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TREATMENT OF THE DISCONTINUITY IN THE SPIN-UP PROBLEM WITH IMPULSIVE START

Raymond Sedney
Nathan Gerber

September 1983

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT COMMAND
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The spin-up equation derived from the Wedemeyer model is considered. For an impulsive start, the boundary conditions prescribe a discontinuity in the velocity on the boundary. The finite difference solution of the spin-up equation is significantly in error unless the discontinuity is treated properly. A solution in the neighborhood of the discontinuity is derived by a direct coordinate expansion, after a stretching transformation is applied. This solution is used as an approximate initial condition; the approximation is evaluated

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I. INTRODUCTION

In the study of the stability of a liquid-filled projectile, it is necessary to determine the flow field of the liquid during the time when rotational motion is imparted to it by the container. This is called the spin-up problem. Typically, the fluid is contained in a right-circular cylinder. In the present discussion, the fluid fills the cylinder. The coordinate system and some notation are shown in Figure 1.

The fluid spins up from rest until it rotates like a solid, except for the flow imparted by the angular motion of the projectile which is neglected in the spin-up problem. Spin-up from rest is an inherently nonlinear problem, but the physics is well understood. The basic work was done by Wedemeyer.[1] This and other spin-up problems are discussed by Greenspan[2] and Benton and Clark.[3] More recently, the spin-up problem has been solved, without the approximations of the Wedemeyer model, using finite difference approximations to the Navier-Stokes equations.[4]

Wedemeyer's model yielded a nonlinear partial differential equation of the diffusion type for the azimuthal velocity,* V, as a function of time and radial coordinate but not axial coordinate. For large Reynolds number, Re, the diffusion terms in this equation can be neglected for some purposes and the solution for V is elementary; Wedemeyer neglected them in his approximate analysis. For other important applications, the diffusion terms must be included giving a second-order, nonlinear, parabolic partial differential equation. The finite difference solutions of this equation, and various aspects of the theory, were presented by Sedney and Gerber.[5]

Two of the main applications of spin-up theory are to the study of the free oscillations, the eigenvalue problem, and forced oscillations arising from the

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**Definitions of symbols are given in List of Symbols section.*

1. *E. H. Wedemeyer, "The Unsteady Flow Within a Spinning Cylinder," U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Report No. 1225, October 1963 (AD 431846). Also Journal of Fluid Mechanics, Vol. 20, Part 3, 1964, pp. 383-399.*

2. *H. P. Greenspan, The Theory of Rotating Fluids, Cambridge University Press, London and New York, 1968.*

3. *E. R. Benton and A. Clark, Jr., "Spin-Up," article in Annual Review of Fluid Mechanics, Vol. 6, Annual Reviews, Inc., Palo Alto, California, 1974.*

4. *C. W. Kitchens, Jr., "Navier-Stokes Solutions for Spin-Up From Rest in a Cylindrical Container," U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02193, September 1979 (AD A077115).*

5. *R. Sedney and N. Gerber, "Viscous Effects in the Wedemeyer Model of Spin-Up From Rest," U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02493, June 1983 (AD A129506).*

angular motion of the projectile, the moment problem. The eigenvalue problem during spin-up, i.e., determination of the frequencies and decay rates of the waves in the rotating fluid, was considered by Sedney and Gerber.[6] This theory was also used to study the spin decay of a liquid-filled projectile[7] after ejection from the gun.

In these applications an accurate solution of the spin-up equation is required. In Reference 5 it was pointed out that one of the sources of error in the finite difference solution was the effect of a discontinuity in the boundary data when the rotation of the container is imparted impulsively. The objectives of this report are to provide the details not included in Reference 5, extend the treatment of this discontinuity, and to give rules for its practical implementation. The Wedemeyer spin-up model, the finite difference method, and the impulsive start assumption are discussed in detail in Reference 5. Most of the notation here is the same as in Reference 5.

II. THE SPIN-UP EQUATION AND BOUNDARY CONDITIONS

The cylinder, with radius a and height 2c, is filled with fluid and initially at rest. At time t = 0, it is given an angular velocity Ω about its axis which remains constant; i.e., the angular velocity history is a step function or Heaviside function. A description of the resulting flow is required. Lengths, velocities, pressure, and time are made dimensionless by a, $a\Omega$, $\rho\Omega^2 a^2$ and Ω^{-1}, respectively. In the inertial frame cylindrical coordinates r, θ, z are used with velocities U, V, W in those directions, respectively; the origin of z is at the center of the cylinder. Dimensionless time is t. Derivatives are indicated by subscripts.

The solution to the spin-up problem is governed by two nondimensional parameters: the aspect ratio c/a and the Reynolds number

$$Re = \Omega\, a^2/\nu$$

where ν is the kinematic viscosity of the liquid. During spin-up boundary layers, called Ekman layers, exist on the endwalls. For Re < 10^5, approximately, the Ekman layers are laminar, whereas they are turbulent for larger Re; obviously, this criterion is by no means precise. The spin-up equation differs for laminar and turbulent Ekman layers.

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6. *R. Sedney and N. Gerber, "Oscillations of a Liquid in a Rotating Cylinder: Part II. Spin-Up," U.S. Army Ballistic Research Laboratory, Aberdeen Proving Ground, Maryland, BRL Technical Report ARBRL-TR-02489, May 1983 (AD A129094).*

7. *C. W. Kitchens, Jr., N. Gerber, and R. Sedney, "Spin Decay of Liquid-Filled Projectiles," Journal of Spacecraft and Rockets, Vol. 15, No. 6, Nov-Dec 1978, pp. 348-354. See also BRL Report No. 1996, July 1977 (AD A043275).*

It is convenient to have a measure of the time scale for the spin-up process and this depends on the state of the Ekman layers. If the Ekman layers are laminar, the characteristic time, in seconds, for spin-up is

$$\bar{t}_s = (2\ c/a)\ Re^{1/2}/\Omega \qquad \text{(sec)}$$

or in nondimensionalized form

$$t_s = (2\ c/a)\ Re^{1/2}.$$

If the Ekman layers are turbulent, the characteristic time for spin-up is

$$\bar{t}_{st} = (28.6\ c/a)\ Re^{1/5}/\Omega \qquad \text{(sec)}$$

or in nondimensionalized form

$$t_{st} = (28.6\ c/a)\ Re^{1/5}.$$

These results are derivable from linear spin-up theory or the Wedemeyer solution without diffusion. The time scales do not give the time for sensible conclusion of the spin-up process; they are a measure of the time for a change of 1/e from the initial state.

The conditions for which the Wedemeyer spin-up model[1] is valid can be presented in terms of the time scales involved. These are the time for one rotation of the container, the spin-up time and the time scale for diffusion across the radius of the cylinder. The model was derived on the basis of an impulsively started cylindrical container, a condition which can only be approximated in practice. Additional time scales must be introduced to assess the approximation to an impulsive start; these are discussed in Reference 5. The acceleration of a projectile in a gun tube is a relatively good approximation to an impulsive start. Of course, the actual time history of projectile angular velocity in the gun tube could be used in the spin-up problem, after suitable modification of the Wedemeyer impulsive start model, but the finite difference calculation would require time steps which would be small compared to the acceleration time, which is typically 0.02 sec, and would require a new calculation for each gun tube twist and for each zone charge. Such an approach is less practical and more time consumming than using the impulsive start approximation.

For an impulsive start, Wedemeyer's model[1] determines the "core flow," i.e., the flow exterior to the Ekman layers and a sidewall (Stewartson) layer. Although the action of the endwalls, through the Ekman layers, is essential to the spin-up process, only the solution to the core flow is determined by the model. Using order-of-magnitude arguments, Wedemeyer showed that the Navier-Stokes equations reduce, for the core flow, to

$$V_t + U\,(V_r + V/r) = Re^{-1}\,[V_{rr} + (V/r)_r] \tag{2.1}$$

and

$$U_z = V_z = P_z = 0 \tag{2.2}$$

plus

$$P_r = V^2/r \quad \text{and} \quad (rU)_r + r\,W_z = 0.$$

For $Re \to \infty$ he proposed neglecting the diffusion terms in (2.1) so that

$$V_{w_t} + U_w\,(V_{w_r} + V_w/r) = 0 \tag{2.3}$$

where the subscript w denotes this approximation.

To solve (2.1) or (2.3), a relationship between U and V is necessary. This is called the Ekman compatibility condition because the Ekman layer suction must be made compatible with the core flow. A phenomenological approach was necessary at this point in the theory. For laminar Ekman layers, Wedemeyer proposed

$$\begin{aligned} U &= -k_\ell\,(r - V) \\ k_\ell &= \kappa\,(a/c)\,Re^{-1/2} = 2\kappa/t_s \end{aligned} \tag{2.4}$$

with $\kappa = 0.443$; $\kappa = 0.5$ often gives results in better agreement with numerical solutions to the Navier-Stokes and will usually be used here.

For turbulent Ekman layers

$$\begin{aligned} U &= -k_t\,(r - V)^{8/5} \\ k_t &= 0.035\,(a/c)\,Re^{-1/5} \approx 1/t_{st}. \end{aligned} \tag{2.5}$$

Using (2.4), (2.3) can be solved for $V\,(r, t)$ with $V\,(r, 0) = 0$, and $V\,(1, t) = 1$:

$$\begin{aligned} V_w &= (re^{2k_\ell t} - 1/r)/(e^{2k_\ell t} - 1) && \text{for } r \geq e^{-k_\ell t} \\ &= 0 && \text{for } r \leq e^{-k_\ell t} \end{aligned} \tag{2.6}$$

Thus $r = r_f = e^{-k_\ell t}$ separates rotating and nonrotating fluid and is called the front. Using (2.5) in (2.3), a numerical integration is necessary but the character of the solution is the same as when (2.4) is used. The solution (pp. 8-16 of Reference 8) is

$$k_t t = \int_r^1 (\xi - y/\xi)^{-8/5}\, d\xi \qquad (2.7)$$

where

$y = r\, V_w$ (y kept constant for the integration).

The solution is completed by obtaining U from (2.4) or (2.5) and W from the continuity equation. The solutions (2.6) and (2.7) have a shear discontinuity, i.e., discontinuous V_{w_r}, which would be smoothed out if the diffusion terms were retained in (2.1).

It follows easily from (2.1) and (2.4) that

$$V = f\,(r,\ k_\ell t,\ k_\ell\, Re);$$

therefore $k_\ell t$ (= t/t_s for $\kappa = 0.5$) is the natural time scale. For the turbulent case k_ℓ is replaced by k_t.

In accordance with (2.2), V is independent of z. To solve (2.1) for V(r, t) initial and boundary conditions must be applied. For the present problem these are:

$$V = 0 \qquad \text{for } t \leq 0, \qquad 0 \leq r \leq 1 \qquad (2.8)$$

$$V = 1 \qquad \text{for } r = 1 \qquad t > 0 \qquad (2.9)$$

$$V = 0 \qquad \text{for } r = 0 \qquad t > 0. \qquad (2.10)$$

The analogy between (2.1) and the heat conduction equation allows one to deduce that these are necessary and sufficient for determining a solution. The form of (2.9) follows from the impulsive start condition. At r = 1, V would be a specified function of t for a nonimpulsive start. Conditions (2.8) and (2.9) require a discontinuity in V at the point r = 1, t = 0.

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8. *Engineering Design Handbook, Liquid-Filled Projectile Design, AMC Pamphlet 706-165, April 1969 (AD 853719).*

A finite difference scheme for solving (2.1) was presented in Reference 5. The value of V (1,0) is required but since $0 \leq V(1,0) \leq 1$ the value is undetermined. If, for example, a value of V (1,0) = 0, 1/2, or 1 is used, there will be a significant error in V; the error is largest near r = 1. Because of the diffusive nature of (2.1), this error will decrease as t increases. However, it was found that the error was often significant even for $t \approx t_s$. Obtaining a finite difference solution when a discontinuity in boundary conditions exists can always be expected to require special treatment. The approach adopted here and presented in Section IV is to obtain a local solution which includes the effects of the discontinuity; that solution is used to generate a new initial condition. Before doing that we digress to consider the manner in which the discontinuity is treated for the nondiffusive equation.

III. THE NONDIFFUSIVE EQUATION

The approximation of (2.1) by (2.3) is clearly a singular perturbation since the second order equation is reduced to a first order equation. The boundary conditions (2.8) and (2.9) are applied to (2.3); (2.10) cannot be imposed but the solution (2.6) satisfies it. The discontinuity can be accommodated in solving (2.3), as will be shown.

It is convenient to introduce the circulation, $\Gamma = r V_w$, and the time scale

$$t' = k_\ell t.$$

Only the laminar case will be considered here. Then (2.3) becomes, using (2.4),

$$\Gamma_{t'} - (r - \Gamma/r) \Gamma_r = 0. \qquad (3.1)$$

This first order equation has a one-parameter family of characteristics and can be solved by the standard method of characteristics. These characteristics contain some information related to the properties of the solution of the original, diffusive equation, (2.1), and they are called the sub-characteristics of the original equation.[9] Along a characteristic of (3.1) $\Gamma = \Gamma_0$ = constant, $0 \leq \Gamma_0 \leq 1$, and the equation of the characteristics is

$$r^2 = \Gamma_0 + (\gamma^2 - \Gamma_0) e^{-2t'}. \qquad (3.2)$$

where γ is a constant, $0 \leq \gamma \leq 1$. This equation can be written in two parts:

9. *J. D. Cole, Perturbation Methods in Applied Mathematics, Blaisdell Publishing Company, Waltham, MA, 1968.*

$$r = \gamma e^{-t'}, \qquad \Gamma_0 = 0 \tag{3.3}$$

which curves issue from $t' = 0$, $0 \leq r \leq 1$ and

$$r^2 = \Gamma_0 + (1 - \Gamma_0) e^{-2t'}, \qquad \gamma = 1 \tag{3.4}$$

which curves emanate from the point $r = 1$, $t' = 0$ where the discontinuity exists. The characteristic common to (3.3) and (3.4) is the front

$$r = r_f = e^{-t'}.$$

Some examples of the characteristics are shown in Figure 2. For the characteristics to the left of the front, (3.3), $r \to 0$ as $t' \to \infty$ whereas, for those to the right, (3.4), $r \to \Gamma_0^{1/2}$ as $t' \to \infty$. Some discussion of the method of characteristics was given by Wedemeyer[8] and Weidman.[10]

The set of characteristics, (3.4), that originates at the point $r = 1$, $t' = 0$ is called a fan. A more familiar example of a fan occurs in supersonic flow over a corner, the Prandtl-Meyer expansion. All the given data, $0 \leq \Gamma_0 \leq 1$, at the discontinuity is convected into $t' > 0$ along the fan. The initial condition $\Gamma_0 = 0$ is convected along (3.3).

Using the method of characteristics provides some insight into the solution of (2.3) and, in particular, how the discontinuity is accommodated. Of course, all the information is contained in (2.6). It could be used to obtain an initial condition for (2.1) to treat the discontinuity. (See Section V.) It would be better than assuming a value for V(1,0) but considerably less accurate than the method discussed next.

IV. THE SOLUTION FOR SMALL TIME

Introducing (2.4) and the time scale $t' = k_\ell t$ into (2.1) gives

$$V_{t'} + (V - r)(V_r + V/r) = \varepsilon\,[V_{rr} + (V/r)_r] \tag{4.1}$$

---

10. *P. D. Weidman, "On the Spin-Up and Spin-Down of a Rotating Fluid: Part 1. Extending the Wedeme~~ Model," Journal of Fluid Mechanics, Vol. 77, Part 4, 1976, pp. 685-7~8.*

where $\varepsilon = 1/k_\ell$ Re. The boundary and initial conditions are still (2.8)-(2.10). The laminar Ekman layer case will be treated in detail; the results for the turbulent case are merely indicated.

The domain of integration for (4.1) is shown in Figure 3a together with the boundary and initial conditions; the discontinuity is at point P: $r = 1$, $t = 0$. A solution is sought which satisfies (4.1), (2.8), and (2.9) in the neighborhood of P. The approach is to stretch the neighborhood of P, by means of a singular transformation, so that the behavior of V can be examined. The coordinates (r, t') are transformed to (R, T):

$$R = (1 - r)/2\,(\varepsilon\, t')^{1/2}$$
$$T = t' \qquad (4.2)$$

This definition of R is $1/2\ \varepsilon^{1/2}$ that of the R in Reference 5. The domain of integration in the transformed plane is shown in Figure 3b with the transformed boundaries of the (r, t') region indicated. The point P transforms into the positive R axis, the initial line $t' = 0$ transforms to the point at infinity, and $r = 0$ transforms into a hyperbola-like curve.

Equation (4.1) is transformed to (R, T) and a solution near $T = 0$ is sought. The form of the equation suggests the expansion

$$V = V_0\,(R) + T^{1/2}\,V_1\,(R) + \ldots \qquad (4.3)$$

where V_0 and V_1 satisfy

$$V_0'' + 2\,R\,V_0' = 0 \qquad (4.4)$$

$$V_1'' + 2\,R\,V_1' - 2\,V_1 = 2\,V_0'\,[\varepsilon^{1/2} + \varepsilon^{-1/2}\,(1 - V_0)] \qquad (4.5)$$

with $' = d/dR$. The series (4.3) is a singular perturbation as evidenced by the fact that the partial differential equation has been replaced by ordinary differential equations and, as will be shown, the boundary condition at $r = 0$ cannot be satisfied. In the terminology of Van Dyke[11] (4.3) is a direct coordinate expansion. Such expansions are usually nonuniform, e.g., diverging for large T which is true for (4.3). For T small enough the two-term series (4.3) gives an adequate approximation to V.

The boundary conditions for V_0 are obtained from (2.8) and (2.9):

$$V_0\,(0) = 1 \qquad V_0\,(\infty) = 0.$$

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11. *M. Van Dyke, Perturbation Methods in Fluid Mechanics, Academic Press, New York, NY, 1964.*

The solution to (4.4) is

$$V_0 = \text{erfc } R. \tag{4.6}$$

For $r \approx 1$, this is the same as the solution for the impulsively started, infinite plate (the Rayleigh problem) which is to be expected since curvature effects are negligible for $r \approx 1$. The boundary conditions for V_1 are

$$V_1 (0) = V_1 (\infty) = 0$$

and the solution to (4.5) is then

$$V_1 = \varepsilon^{1/2} R (1 - \text{erf } R) + \varepsilon^{-1/2} [R (1 - \text{erf}^2 R) - \pi^{-1/2} e^{-R^2} \text{erf } R]. \tag{4.7}$$

The one-term solution, $V = V_0 = \text{erfc } R$ has the property that V = constant along the parabolas in the (r, t) plane, R = constant. This is somewhat analogous to the property of the nondiffusive equation that Γ = constant along the fan (3.4).

The original objective was to obtain a solution to (4.1) in the neighborhood of P but (4.3), (4.6), and (4.7) predict V for all r. For $r = 0$, or $R = 1/2\ (\varepsilon T)^{1/2}$, it is clear that neither V_0 nor V_1 is zero so that (2.10) is not satisfied. In principle a solution near $r = 0$ should be obtained and matched to (4.3). In practice, for application to the finite difference problem this is not necessary, as shown in the next section.

Plots of V_1 for three values of ε and V_0 are presented in Figure 4. Both V_0 and V_1 decrease exponentially as $R \to \infty$. Their asymptotic forms are

$$V_0 \sim (1/\sqrt{\pi})\ e^{-R^2}/R \tag{4.8}$$

$$V_1 \sim (1/\sqrt{\pi})\ (\varepsilon^{1/2} + \varepsilon^{-1/2})\ e^{-R^2} . \tag{4.9}$$

For estimation purposes (4.8) is in error by 1 in the second decimal place for $R = 1.34$ and by 1 in the sixth decimal place for $R = 3.12$; for a moderately small $\varepsilon = .0313$, corresponding to $Re = 4 \times 10^4$ and $c/a = 3.120$, (4.9) is in error by 1 in the first decimal place for $R = 1.34$ and by 11 in the fifth decimal place for $R = 3.12$.

V. APPLICATION OF THE SOLUTION FOR SMALL TIME

If the initial condition (2.8) is applied at $t = 0$, the difficulties discussed in Section II arise. The limit of (4.3) as t' or $t \to 0$ is (2.8). Therefore, we must apply (4.3) at a small time, the natural choice being one time step, Δt, in the finite difference method. Although we shall make that choice, the method is not restricted to it.

Either the one-term or two-term solutions in (4.3) can be used. We set

$$V = V_0\ (R) \tag{5.1}$$

or

$$V = V_0\ (R) + T^{1/2}\ V_1\ (R) \tag{5.2}$$

at $t = \Delta t$ or $T = k_\ell\ \Delta t$. Expressing R in terms of the original variables gives

$$R = (1 - r)\ (Re/\Delta t)^{1/2}/2.$$

The functions of r computed from either (5.1) or (5.2) provide the initial conditions at $t = \Delta t$. The fact that $V \neq 0$ at $r = 0$ is of no practical concern. Only an approximate initial condition can be expected and the approximation at $r = 0$ is quite good. For example, if $R \geq 3.5$, $V < 10^{-6}$ if (5.1) is used; for a range of Δt and ε the same is true if (5.2) is used. For $r = 0$ this condition on R is

$$(Re/\Delta t)^{1/2} \geq 7. \tag{5.3}$$

For $\Delta t = 1$, the lower bound on Re is 49. For smaller Re, Δt must be decreased.

There is an independent test of the permissible Δt in the iterative finite difference calculation. The Δt is required to be small enough so that no more than 4 iterations are required for convergence at any time step. Experience has shown that if the Δt from the criterion is satisfied, (5.3) is also. The requirement on Δr for the finite difference calculation is based mainly on experience and/or tests; see Reference 5 for a discussion of this. There is obviously an interaction between the Δr error in the finite difference scheme and the accuracy of the initial condition. For small enough Δt, (5.2) is more accurate than (5.1), and using it gives a more accurate V for a given Δr; alternatively, for a given accuracy, a larger Δr can be used with (5.2).

The accuracy of the two-term solution, for small t, can be illustrated by comparing it with the finite difference solution of (2.1) using (2.4); a criterion for small t is given in the next paragraph. For Case 1, with Re = 4×10^4, c/a = 3.120, and ϵ = .0313, the solution V (r, t) is shown in Fig. 5 for $.84 \leq r \leq 1.0$ at t = 40 with initial condition (5.2) at $t = \Delta t = 5$. The solutions V_0 and $V_0 + T^{1/2} V_1$ at t = 40 are plotted and show that the two-term approximation is still accurate at t = 40. The nondiffusive solution, (2.6), is also plotted to illustate the remark at the end of Section III.

In the discussion after (4.3) the likelihood that it would diverge for increasing T was mentioned. This is illustrated in Figure 6. The finite difference result for V, using (5.2) at $\Delta t = 5$, is shown vs t for Case 1, r = .94 and Case 2, r = .995. Case 2 parameters are Re = 4×10^5, c/a = 1, and ϵ = .00316. The one-term and two-term solutions are also shown. For Case 1, $V_0 + T^{1/2} V_1$ is a good approximation to V but is beginning to diverge from it. For Case 2, the divergence is quite evident and in fact $V_0 + T^{1/2} V_1 \geq 1$ for $t \geq 42$. Since V > 1 is impossible, the two-term solution is not valid near $t = 42$. An approximate expression for the time of breakdown can be obtained from the condition V_r (r = 1) = 0. Using the two-term approximation for V this gives

$$(V_0 + T^{1/2} V_1)_r|_{r = 1} = 0.$$

Using (4.6) and (4.7) this yields

$$T^{1/2} [\epsilon^{1/2} + \epsilon^{-1/2} (1 - 2/\pi)] = 2\pi^{-1/2}$$

or, for ϵ small,

$$T \cong 10\epsilon \tag{5.4}$$

as the approximate time of breakdown. For Case 1 in Figure 6, this gives $t \simeq 400$ and for Case 2 $t \simeq 40$. If the solution is valid up to a time which is some fraction, C, of T in (5.4)

$$t \cong (10\ C/\kappa^2)\ (c/a)^2$$

as a practical upper-limit for the validity of the two-term approximation. For the cases considered C = 0.5 is suitable.

The crucial tests of the application of (5.1) or (5.2) is the accuracy of the finite difference solution for times comparable to t_s. Results will be shown for Case 3 with Re = 4974, c/a = 3.30, ε = .1056, and κ = .443 giving = 525. The comparison will be given first in terms of the vorticity $\zeta = (r\,V)_r/2r$. This is an important quantity because it occurs, along with V, in the perturbation equations. For comparison purposes, it has the advantage that an exact result is known: ζ_r (r = 1) = 0 for all t. This follows directly from (4.1). In Figure 7 ζ vs r is shown for Case 3 at t = 100 and 600; ζ is obtained from the finite difference solution using (5.1) as initial condition at Δt = 5 and using V (1,0) = 1. For t = 100 the errors resulting from use of V (1,0) = 1 are quite large; for t = 600 the errors are smaller but would be unacceptable in the solution of the eigenvalue problem. For t = 100, ζ_r (r = 1) = -240 for the V (1,0) = 1 curve; ζ_r (r = 1) = -1.5 for the curve using (5.1). The results using (5.2) are not shown in Figure 7 but ζ_r (r = 1) = 0.32 is obtained.

For t = 100, the assumption V (1,0) = 1 causes errors in V of the order of 10%; whereas, for t = 600 the errors are about 3%. Experience has shown that errors of 3 - 4% in V give erroneous results for the eigenvalue.

VI. THE TURBULENT CASE

The same kind of approach can be used to derive the initial condition for the turbulent case. In fact the one-term solution is the same as for the laminar case, (5.1). Only the essential steps will be given here. For the turbulent case, (2.1) with (2.5) must be solved for V with the same boundary conditions (2.8) - (2.10). Analogous to (4.1), V satisfies

$$V_{t'} - (r - V)^{8/5}\,(V_r + V/r) = \varepsilon_t\,[V_{rr} + (V/r)_r] \qquad (6.1)$$

where now $t' = k_t t$ and $\varepsilon_t = 1/k_t\,\mathrm{Re}$. Again the transformation (4.2) and the expansion (4.3) are used. The equation for V_0 and its solution are the same as (4.4) and (4.6), respectively. The equation for V_1 is now

$$V_1'' + 2\,R\,V_1' - 2\,V_1 = 2\,V_0'\,[\varepsilon_t^{1/2} + \varepsilon_t^{-1/2}\,(1 - V_0)^{8/5}] \qquad (6.2)$$

With V_0 = erfc R it is unlikely that a closed form solution to (6.2) can be obtained as was done for the laminar case; the homogeneous solution of (6.2) is the same as before. A numerical integration of (6.2) is certainly feasible but has not been incorporated into the initial condition. For the turbulent case only the one-term solution, V_0, has been used.

VII. CONCLUSIONS

For an impulsive start, the boundary conditions for the spin-up equation prescribe a discontinuity in the velocity on the boundary. If this discontinuity is not treated properly, the finite difference solution of the spin-up equation will be in error. Although the errors continually decrease (because the spin-up equation is of the diffusion type) they may be significant even for one spin-up time. The treatment of the discontinuity proposed here consists of determining a local solution near the discontinuity and using that as an approximate initial condition. The degree of approximation has been investigated and limits of applicability determined. Tests of this approximate initial condition, over a range of Re and c/a, have shown that the errors in the solution are negligible at about one-half of the spin-up time; the time for negligible errors can be decreased by adjusting the parameters of the initial solution.

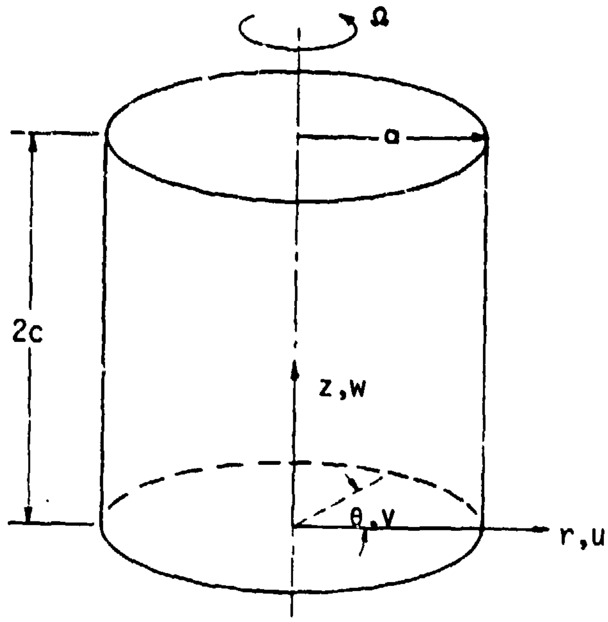

Figure 1. Cylinder, Coordinates, and Notation.

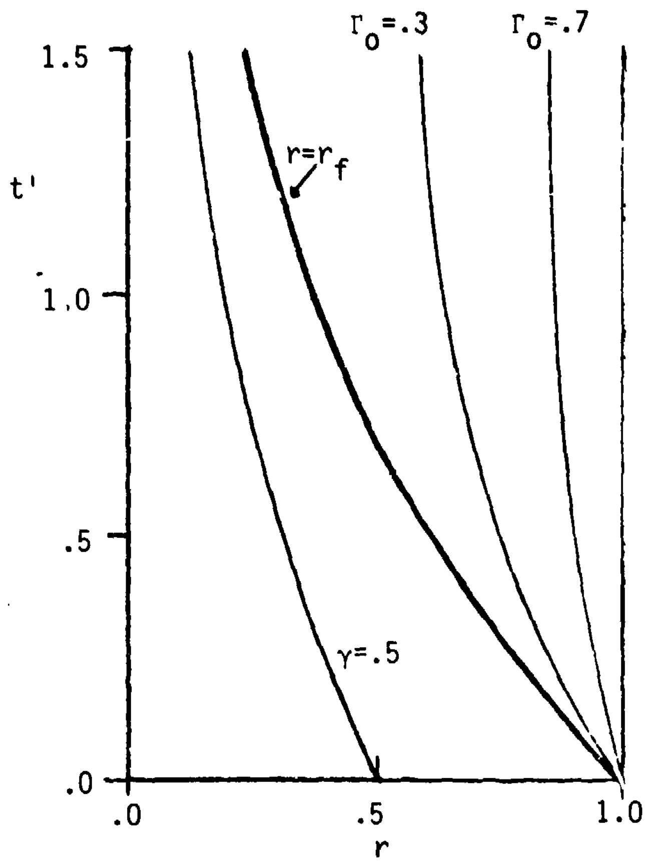

Figure 2. Characteristic Curves for (3.1). Numerical Values of the Parameters are for (3.3) and (3.4).

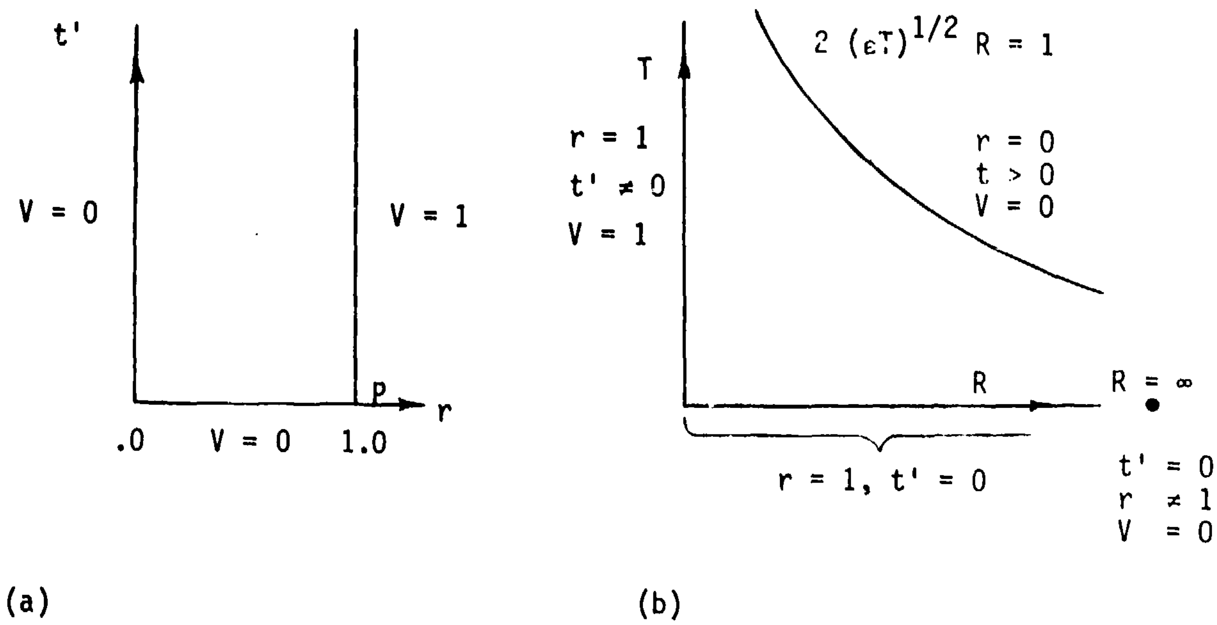

Figure 3. Domain of Integration and Boundary Conditions for (4.1) in (a) the (r, t) Plane and (b) the Transformed (R, T) Plane; See (4.2).

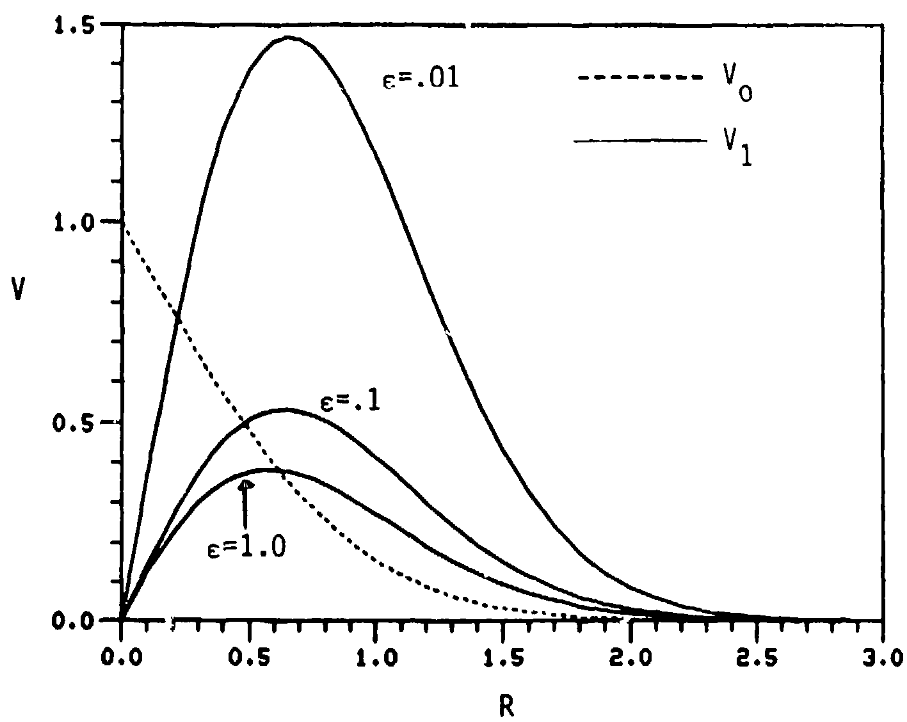

Figure 4. V_1 vs R for Three Values of ε and V_0; See (4.6) and (4.7).

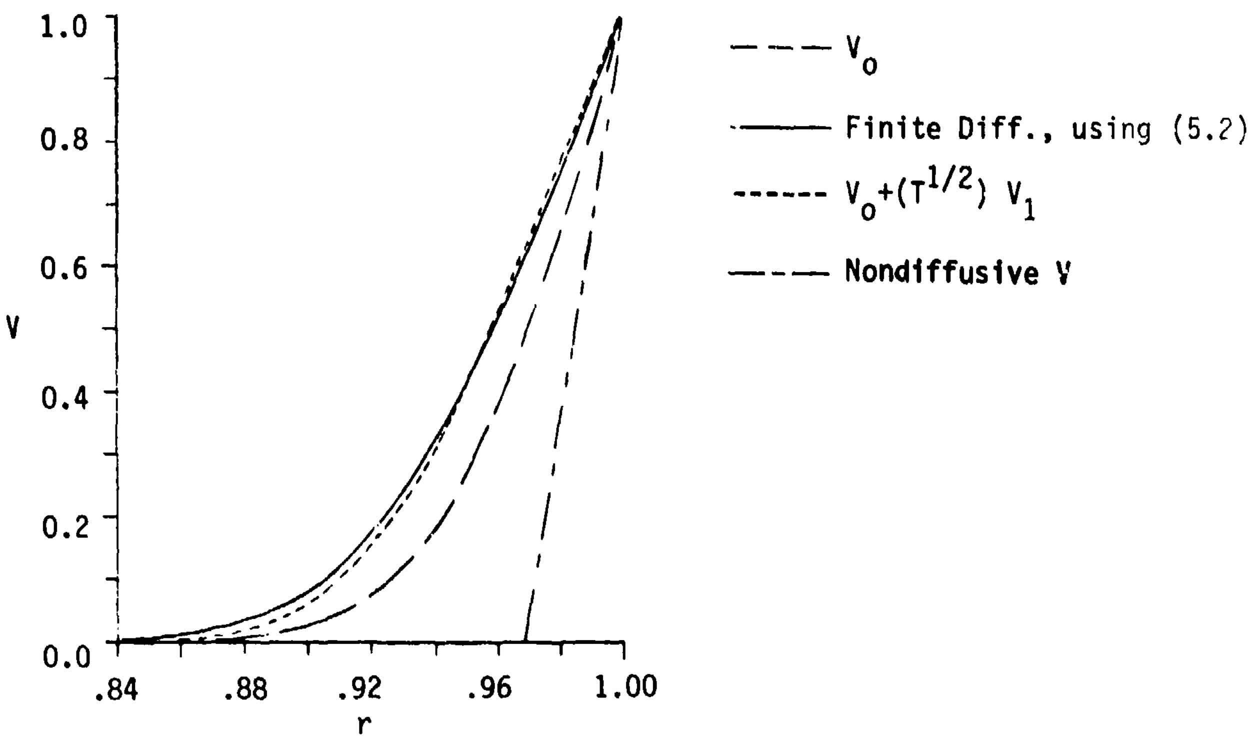

Figure 5. Comparison of the Finite Difference Solution for V at t = 40 With the One-Term and Two-Term Solutions, (5.1) and (5.2), and the Nondiffusive Solution (2.6).
Re = 4×10^4, c/a = 3.120, ε = 0.0313.

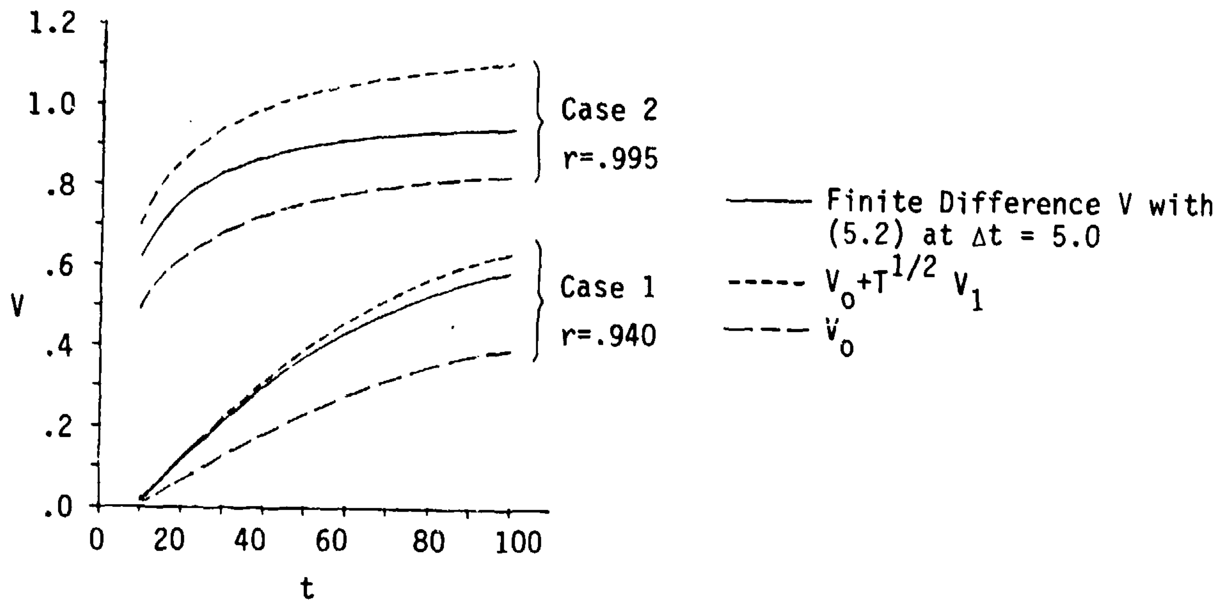

Figure 6. Finite Difference Solution V vs t for Cases 1 and 2 Compared With the One-Term and Two-Term Approximate Solutions.
Case 1: Re = 4×10^4, c/a = 3.120;
Case 2: Re = 4×10^5, c/a = 1.

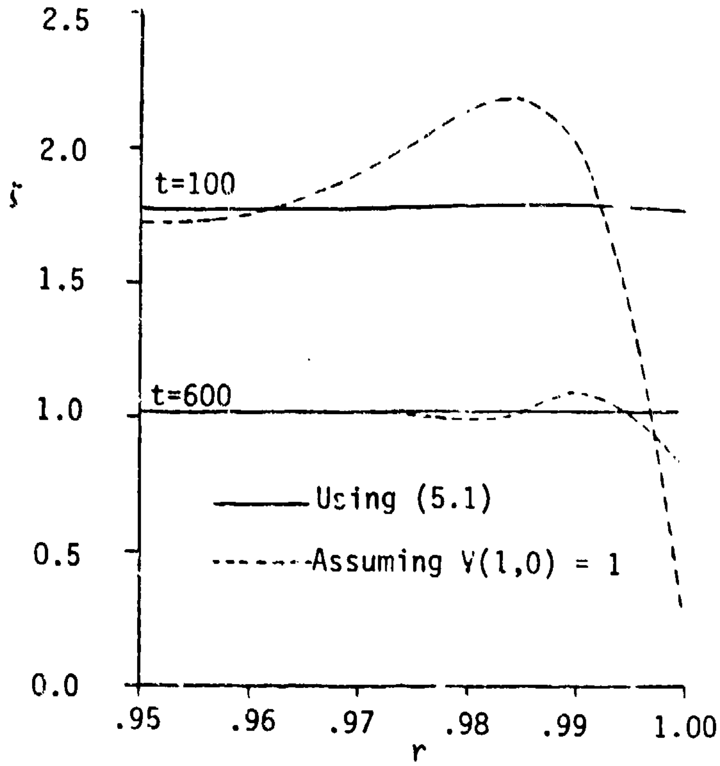

Figure 7. Vorticity ζ vs r for Re = 4974, c/a = 3.30, ε = 0.1056 at t = 100 and 600 Assuming V(1,0) = 1 or Using (5.1) at Δt = 5.

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LIST OF SYMBOLS

a	cross-sectional radius of cylinder
c	half-height of cylinder
k_ℓ	$\equiv \kappa\,(a/c)\,Re^{-1/2}$ (See (2.4).)
k_t	$\equiv 0.035\,(a/c)\,Re^{-1/5}$ (See (2.5).)
N	number of subintervals in r in finite-difference solution
P	pressure/$(\rho a^2 \Omega^2)$ in Wedemeyer model (See (2.2).)
r	radial coordinate/a
r_f	location of front in Wedemeyer nondiffusive model
R	$\equiv (1 - r)/(4\varepsilon t')^{1/2}$
Re	$\equiv a^2\Omega/\nu$, Reynolds number
t	time $\times\ \Omega$
t_s	$\bar{t}_s\ \Omega$
t_{st}	$\bar{t}_{st}\ \Omega$
t'	$\equiv k_\ell t$ (or $k_t t$)
$\bar{t}_s$	$\equiv (2\ c/a)\ Re^{1/2}/\Omega$, characteristic spin-up time for laminar Ekman layer
$\bar{t}_{st}$	$\equiv (28.6\ c/a)\ Re^{1/5}/\Omega$, characteristic spin-up time for turbulent Ekman layer
T	$= t'$ (see (4.2))
U, V, W	radial, azimuthal, axial velocity components $\times\ 1/(a\Omega)$ of Wedemeyer model spin-up flow with diffusion (See (2.1) and (2.2).)
U_w, V_w	radial and azimuthal velocity components $\times\ 1/(a\Omega)$ of Wedemeyer model spin-up flow without diffusion (See (2.3).)

$V_0(R)$, $V_1(R)$	zeroth and first order coefficients of $T^{1/2}$ in (4.3)
z	axial coordinate ($z = 0$ at cylinder midplane.)
γ	constant in characteristic equation, (3.2)
Γ	$\equiv rV_w$, circulation
Γ_0	constant in characteristic equation, (3.2)
Δt	t-interval size in finite difference solution
ε	$1/(k_\ell Re)$
ε_t	$1/(k_t Re)$
ζ	$\equiv (rV)_r/(2r)$, nondimensional vorticity
θ	azimuthal angle
κ	constant in expression for radial velocity with laminar Ekman layer (See (2.8).)
ν	kinematic viscosity of fluid
ρ	density of fluid
Ω	angular velocity of spinning cylinder

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